**Lecture 3.**

# Analytic geometry in the plane.
## Cartesian coordinate system in the plane.The lines of the first order.

Rectangular Coordinate System is called as Cartesian coordinates system. Each point in the plane is assigned a pair of numbers called the coordinates of the point.

- To locate points in a plane, two perpendicular lines are used: a horizontal line called the ***x*-axis** and a vertical line called the ***y*-axis.**
- The *x*-axis, the *y*-axis, and all the points in their plane are called a **coordinate plane.**
- The point of intersection of the *x*-axis and the *y*-axis is called the **origin.** The origin is denoted by O with coordinates (0, 0).
- The number to the left of the comma in an ordered pair is the ***x*-coordinate** of the point and indicates the amount of movement parallel to the *x*-axis from the origin. The movement is to the right if the number is positive and to the left if the number is negative. On the x-axis, values to the right are positive and those to the left are negative.
  The number to the right of the comma in an ordered pair is the ***y*-coordinate** of the point and indicates the amount of movement perpendicular to the *x*-axis. The movement is above the *x*-axis if the number is positive and below the *x*-axis if the number is negative. On the y-axis, values above the origin are positive and those below are negative.
- The *x*-axis and *y*-axis separate the coordinate plane into four regions called **quadrants.** The upper right quadrant is quadrant I, the upper left quadrant is quadrant II, the lower left quadrant is quadrant III, and the lower right quadrant is quadrant IV.
- The location of the point A in a plane is determined by two coordinates, written as an ordered pair, ( x,y). The first coordinate x of the point A is defined by dropping perpendicular on the axis Ox, the second coordinate y is defined by dropping perpendicular on the axis Oy.

The distance between two any points with coordinates $(x_1, y_1)$ and $(x_2, y_2)$ in the plane is given by formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

It is obtained with the help of the Pythagoras' Theorem

## A Straight Line

Equation of a straight line has the next different forms:

1. Equation of a line with slope - **Slope-Intercept Form of a Straight Line**

$$y = mx + b,$$

where $m$ is the slope of this line, b is the $y$-intercept of the line.

Parallel lines have the same **slope**.
Perpendicular lines have slopes that the its product is equals −1.

Slope or gradient of a Line may be calculated by the formula

$$m=(y_2-y_1)/(x_2-x_1)$$

**2. Point-Slope Form of a Straight Line -** equation of the line passing through the point $(x_1, y_1)$, with the slope $m$
We use this form when we need to find the equation of a line passing through a point $(x_1, y_1)$ with slope $m$:

$$y - y_1 = m(x - x_1)$$

**3. Equation of the line passing through two given points**
with coordinates $\mathbf{(x_1, y_1)}$ **and** $\mathbf{(x_2, y_2)}$

$$y - y_1 = ((y_2 - y_1)/\ (x_2 - x_1))/\ (x - x_1)$$

If $y_2$ isn't equal to $y_1$ , this equation is rewritten as

$$y - y_1 /(y_2 - y_1)= (x - x_1)/\ (x_2 - x_1)$$

**4. General Form of a Straight Line**

$$Ax + By + C = 0$$

It can be useful for drawing lines by finding the $y$-intercept when $x$=0 and the $x$-intercept if $y$=0.

**5. Equation of a line in pieces.** Let A,B,C aren't equal to zero. We can transform general equation $Ax + By + C = 0$ into the form x/(-C/A)+y/(-C/B)=1.
Denoting a=-C/A and b=-C/B, we obtain

$$\mathbf{x/a+y/b=1}$$

**6. The normal form of the equation of a straight line:**

$$\mathbf{X\cos\alpha+y\sin\alpha-p=0}$$

$\mu Ax + \mu By + \mu C = 0$. If μ: μA=cosα, μB=sinα, μC=-p we obtain normal form from general form
$\mu=\pm 1/\sqrt{(A^2+B^2)}$   μ - normalization factor

## **Perpendicular Distance from a Point to a Line**

The distance from a point $(m, n)$ to the line $Ax + By + C = 0$ is given by:

$$d = \frac{|\, Am + Bn + C \,|}{\sqrt{A^2 + B^2}}$$